cult to construct. A very good compromise can be found for C_2 between $0.014 < C_2 < 0.04$. This results in $1.04 < |E_z|_{\max} < 1.36$.

Other important properties are the attenuation factor and the Q factor. Taking into account finite conductivity of the waveguide walls for a TM mode, the attenuation constant has the following expression:

$$\alpha = \frac{(C/2A)[(\epsilon_0/\mu_0)]^{1/2}R_m}{[1 - (\omega_c^2/\omega^2)]^{1/2}}\,\xi_c \tag{15}$$

where

- C circumference of the cross section;
- A area of the cross section;
- R_m surface resistance;
- ξ_c dimensionless number depending on the shape of the cross section and given by the following relation:

$$\oint_c \frac{1}{\omega_c^2}\left|\frac{\partial E_z}{\partial n}\right|^2 dl = \xi_c \epsilon_0 \mu_0 \frac{C}{A}\int_A |E_z|^2\, dA. \tag{16}$$

The relative attenuation factor compares the attenuation in the perturbed waveguide by the use of nonseparable solutions, with that of the nonperturbed or rectangular waveguide and is given as follows:

$$\alpha_r = \frac{(C/2A)\xi_c}{(C'/2A')\xi_c'} = [(C/2A)]_r \xi_{cr}. \tag{17}$$

In Fig. 7 we show $(C/2A)_r$ and ξ_{cr} versus the perturbation factor C_2.

It is obvious that only the factor ξ_{cr} has a considerable variation and is therefore a measure of the attenuation factor presented in Fig. 8. For cylindrical cavities, the Q factor is given by:

$$Q = \frac{\mu}{\mu_c}\frac{d}{\delta}\frac{1}{2[1 + \xi_c(Cd/4A)]} \tag{18}$$

where

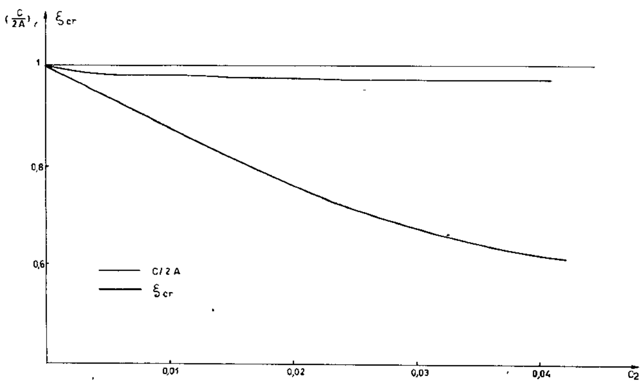

Fig. 7. Relative values of $(C/2A)_r$ and ξ_{cr} as a function of C_2.

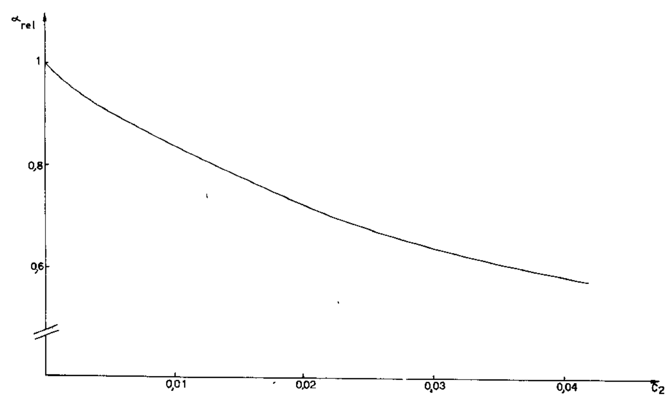

Fig. 8. Relative attenuation factor versus the perturbation factor C_2.

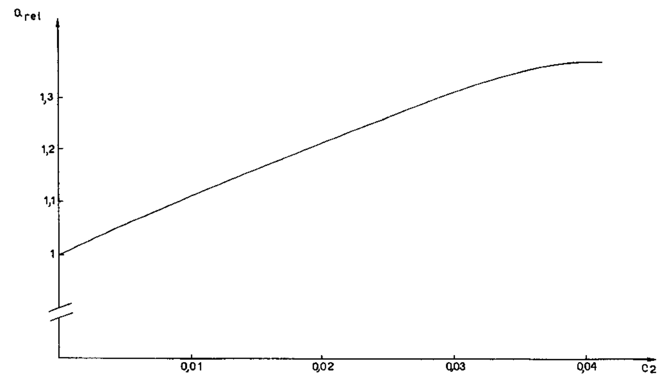

Fig. 9. Relative Q factor versus the perturbation factor C_2.

- u_c permeability of the metal walls of the cavity;
- δ skin depth;
- d length of the cavity.

In the same manner a relative Q factor can be defined in the form

$$Q_r = \frac{1 + \xi_c Cd/4A}{1 + \xi_c' C'd/4A'}. \tag{19}$$

Fig. 9 shows Q_r as an increasing function of the perturbation factor C_2.

In conclusion we can say that nonseparable solutions of the Helmholtz wave equation are suitable for describing TE and TM modes in waveguides and cavities with general cross sections; moreover, they are exact wave functions for any deformed conventional rectangular or circular waveguide so that no approximate method of solutions has to be taken. Further, it has been shown that waveguides and cavities synthesized with nonseparable solutions have better attenuation and higher Q factor than comparable conventional waveguides shapes.

Further, it is clear that they have interesting properties for microwave measurement and power applications. All calculations were done on the computer of the Rekencentrum of the Catholic University of Louvain.

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Tapered Asymmetric Microstrip Magic Tee

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Abstract—The design, development, and constructuion of a very compact decade-(1–10-GHz) bandwidth microstrip −8.34-dB coupler are described. Calculations are given for the voltage coupling coefficient and the low-frequency cutoff, and the method of determing the physical dimensions of the circuit is described. Also, the feasibility of a decade-bandwidth microstrip magic tee by cascading two −8.34-dB couplers is demonstrated by comparing the actual and theoretical results of a coupler.

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INTRODUCTION

This short paper describes the design of an asymmetric broad-band magic tee. The high-pass nature of this coupler makes it inherently broad band, and such a component is applicable for use in many broad-band microwave assemblies including balanced mixers and beam forming networks. Alternate magic tees which have been realized in microstrip have not had the inherent bandwidth capability. Both the slotline/microstrip magic tee [1] and the magic tee using cascade-tandem connection of directional couplers [2] are limited to 2:1 frequency bandwidths.

To validate the design approach, a test circuit was made which required the development of a fine-line delineation process and special packaging techniques to suppress spurious modes. The actual results correspond with the theory except for the phase shift below the minimum frequency of operation. This effect has not been explained. While the results obtained at this time indicate further work is required, the feasibility of such a network has been demonstrated.

The theory for asymmetric couplers has been described by Arndt [3] and Tresselt [4]. Also, Duhamel and Armstrong [5] demonstrated this magic tee in low dielectric stripline transmission media. However, prior to this work the feasibility of producing this coupler in a microstrip configuration had not been demonstrated. This was due to several limitations. First, the planar structure with edge coupling made it difficult to obtain the required tight coupling. Second, the fabrication techniques for achieving repeatable 0.0002-in gaps were not available for a thick metallization layer ($\approx$0.0005 in). Third, the inhomogeneous transmission media with a high dielectric constant substrate degraded the directivity of such couplers because the even- and odd-mode velocities of propagation were not equal. The potential solutions to these problems are described.

The asymmetric microstrip magic tee consists of two -8.34-dB couplers cascaded in tandem to form a -3-dB coupler [6]. Reference lines are added to each side of the coupler to compensate for the frequency-dependent length and to give a transmission phase difference of 180 and 0° at the output ports when fed from the main port and isolated port, respectively. The -8.34-dB coupling was selected because the edge coupling required at the tight coupling region was considered within reason; subsequent processing development resulted in an additive process which achieved the required coupling. Improved directivity was achieved by positioning the top ground plane very close to the dielectric substrate; from the literature [7] 0.025 in was selected as the spacing. However, it was determined empirically that 0.015 in was superior for our coupler dimensions.

ANALYSIS OF -8.34-dB COUPLER

The schematics of the asymmetric high-pass -8.34-dB directional coupler are shown in Fig. 1. They consist of two transmission lines which are coupled nonuniformly along the coupling section l. The voltage coupling coefficient $k(z)$ changes continuously along the longitudinal direction Z. It is defined by even- and odd-mode impedances Z_{0e} and Z_{0o} in (1) as

$$k(Z/l) = \frac{Z_{0e}(Z/l) - Z_{0o}(Z/l)}{Z_{0e}(Z/l) + Z_{0o}(Z/l)}. \tag{1}$$

It is assumed that the four ports of the coupler are terminated in a characteristic impedance Z_0, that the relationship defined by (2) holds along the coupling section, and that the even- and odd-mode velocities are equal. Hence the characteristic impedance of the coupling region Z_0 is given by (2) as

$$Z_{0e}(Z/l) \cdot Z_{0o}(Z/l) = Z_0^2. \tag{2}$$

The voltage coupling coefficient k and the physical length of the coupling section l for the coupler are determined from (3) and (4).

$$l = (l/\lambda)_c \frac{\lambda_{0\,\max}}{(\epsilon_r)^{1/2}} \tag{3}$$

$$k(Z/l) = \sum_{m=0}^{6} K_m (Z/l)^m \tag{4}$$

where ϵ_r is the relative effective permittivity of the media and $\lambda_{0\,\max}$ is the desired maximum free-space wavelength. $(l/\lambda)_c$ and K_m are the low-end cutoff frequency and the coupling factor coefficients, respectively. For a given mean coupling value and voltage ripple δ, K and $(l/\lambda)_c$ can be obtained from Arndt's table [3]. For a -8.34-dB coupler with a 0.5-percent voltage ripple, the low cutoff frequency $(l/\lambda)_c$ is to be 0.409 and the coupling factor coefficients are 0.6687, -1.0507, -0.7719, 2.3164, -1.0692, -0.3929, and 0.3044.

To calculate the impedances and dimensions, the total length of the coupler is divided into 200 equal-length sections, and the voltage coupling coefficient $k(z)$ at the midpoint of each section is determined using (4). Then the even- and odd-mode impedances that correspond to these $k(z)$ values are calculated from (1) and (2) and the physical circuit dimensions are determined using the computer program subroutine provided by Smith [8] for computation of even- and odd-mode fringing capacitances of coupled microstrip lines in suspended substrates. Alumina ($\epsilon_r = 10$) was chosen as the high-dielectric substrate material, and the substrate thickness was chosen to be 0.025 in. The effective dielectric constant ϵ_{eff} with the close proximity ground-plane spacing was equal to 5.5. Then from (3) the length of the coupler was calculated to be 2.05 in for a low-frequency cutoff of 1 GHz.

The circuit was fabricated on a 0.025-in-thick alumina substrate with a top ground plane separated from the circuit by an air space of 0.025 in. The gap width tapers down from 0.070 in at the loosely coupled end to 0.00018 in at the tightly coupled end. The -8.34-dB coupler was tested initially on an automatic network analyzer. Satisfactory results were obtained over a 5:1 frequency bandwidth. However, better performance had been anticipated. Therefore the -8.34-dB coupler was tested on a time-domain reflectometer which measures the coupling amplitude along the coupled lines. Coupling was observed beyond the actual coupled section. It is believed that this undesirable coupling was caused by the surface waves and trapped waveguide modes, which can occur in the rectangular package. To correct this undesirable condition, the width of the circuit package was reduced from 1 to 0.15 in. After this change had been made, the measurements were repeated and it was observed that the overall response of the coupler was improved. The directional coupler is shown in Fig. 2 with top cover removed. Fig. 3 shows the theoretical and the experimental results of the coupled and transmitted signal

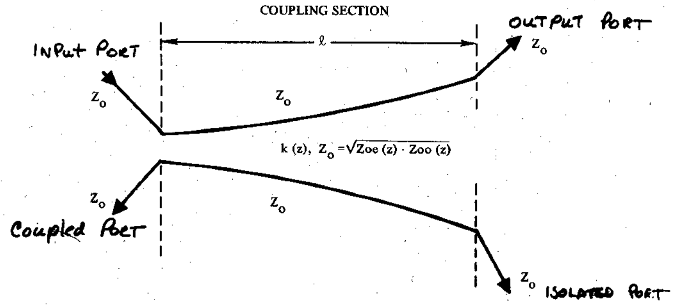

Fig. 1. High-pass transmission-line directional coupler.

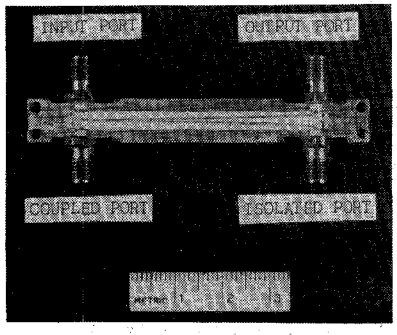

Fig. 2. Tapered asymmetric directional coupler.

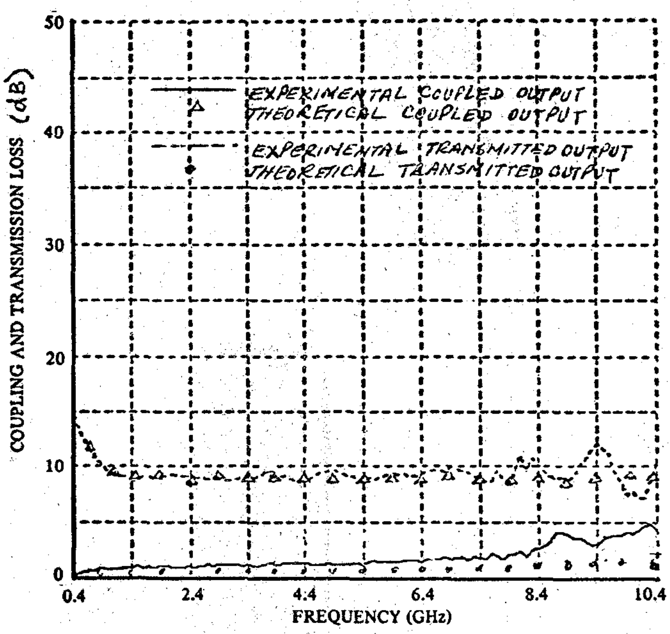

Fig. 3. Coupling and direct loss of −8.34-dB coupler.

amplitudes as a function of frequency. The theoretical analysis is performed by evaluating the cascade *ABCD* matrix of the line segments as a function of frequency. The isolation and the return loss of the coupler (not shown here) were greater than 15 dB over the entire band.

MAGIC TEE CONSTRUCTION

The magic tee was to be constructed by cascading two −8.34-dB asymmetric couplers in tandem and adding phase reference lines on each side of the device. However, the output port and the isolated port of the −8.34-dB coupler had to be reversed in order to solve the topological layout problem. This was achieved by having the coupled lines cross over at the tightly coupled region by using a gold ribbon bridge 1 mil thick and 7 mil wide (see Fig. 4 which gives a detailed view of the crossover region). In making this crossover the bridging ribbon was kept high enough to prevent coupling between adjacent circuit lines. Also, a small channel was cut in the top ground plane in order to provide adequate space between the ribbon and the top ground plane. This was necessary because it was observed that if the bridging ribbon is too close to the coupled lines or to the

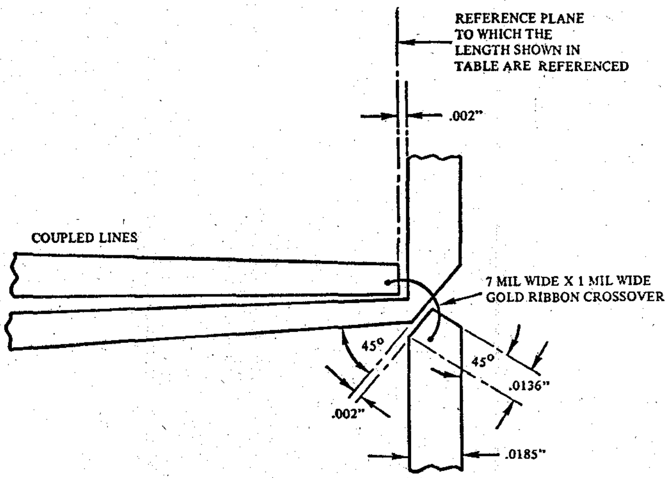

Fig. 4. Detailed view of crossover at the tightly coupled region.

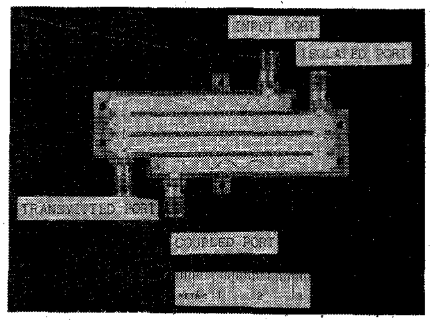

Fig. 5. Tapered asymmetric magic tee.

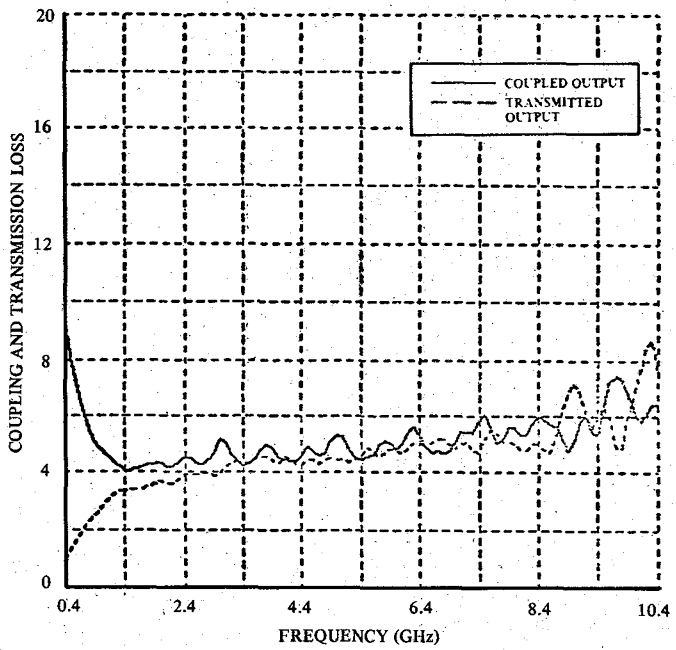

Fig. 6. Coupling and transmission loss of magic tee.

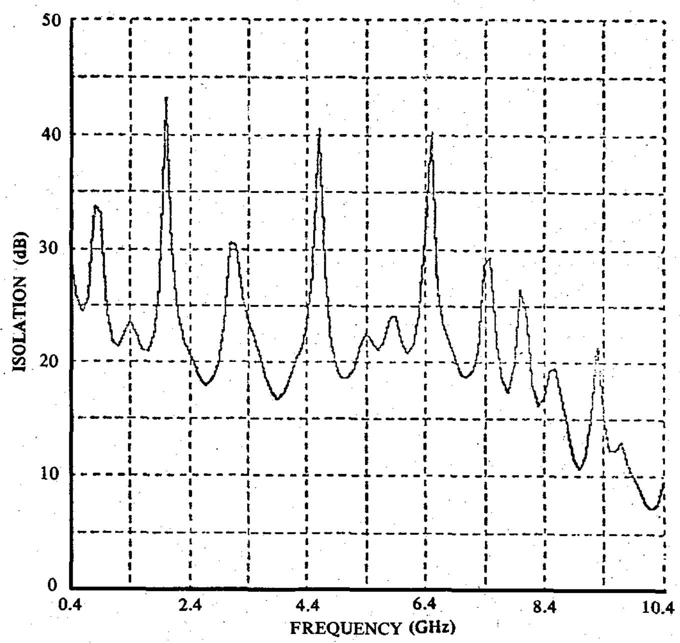

Fig. 7. Isolation of magic tee.

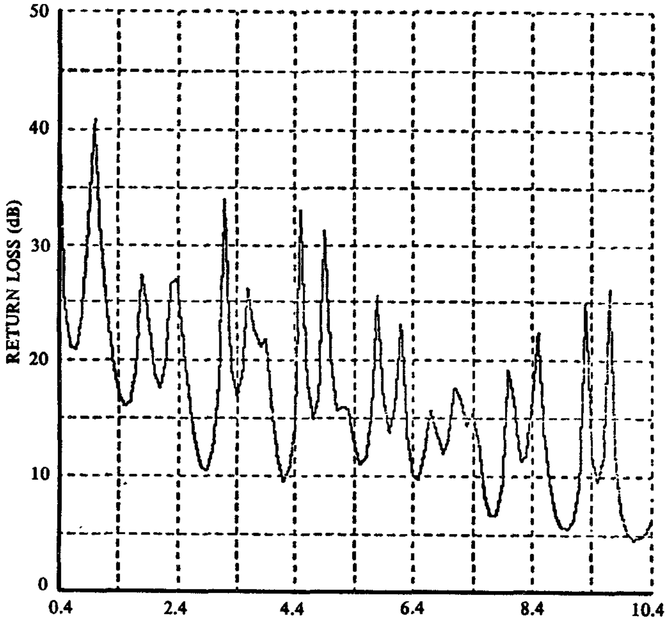

Fig. 8. Return loss of magic tee.

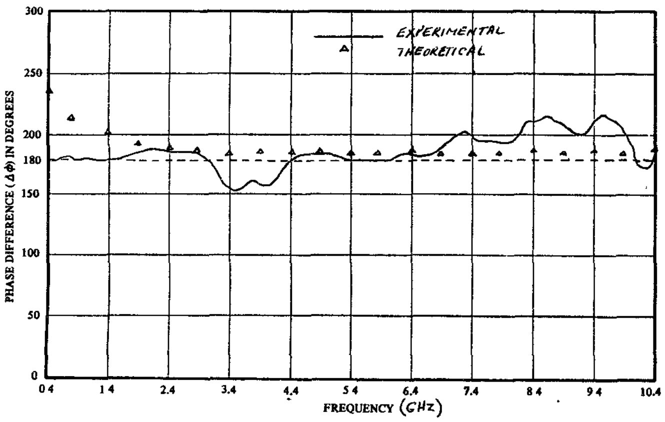

Fig. 9. Phase performance of tapered asymmetric magic tee.

top ground plane, undesirable ripples are induced in the coupling. Also, each coupler section and reference line was made on individual substrates and metal walls separated the substrates in order to reduce spurious modes.

The magic tee circuit designed for a low-frequency cutoff of 1 GHz is shown in Fig. 5 with the top ground plane removed. The actual characteristics of this device are shown in Figs. 6–9. Fig. 6 compares power at the two equal-power output ports. The average power deviation is less than 0.5 dB over the 1–9-GHz frequency band. Fig. 7 shows the isolation and Fig. 8 shows the return loss. Fig. 9 depicts the measured and the computed phase difference between the output ports of the magic tee.

CONCLUSION

This work has demonstrated the feasibility of designing a decade-wide magic tee, using microstrip and a top ground plane. The coupling factor $k(z)$, which changes continuously along the length of the coupler, was calculated for this device using the coupling coefficient prepared by Arndt for equal-ripple high-pass directional couplers. The line and gap widths were determined by using the computer subroutine of Smith. The coupled sections and the reference lines were separated by metal walls in order to reduce the standing-wave modes in the box.

ACKNOWLEDGMENT

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Evaluation of the Equivalent Circuit Parameters of Microstrip Discontinuities Through Perturbation of a Resonant Ring

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***Abstract*—A resonant technique for evaluating the equivalent circuit of reciprocal microstrip discontinuities is described. The complex Z parameters of a discontinuity are related to the change in resonant frequencies and Q factors of a microstrip ring it perturbs. As an example, measurements made on inductive posts are presented and compared with theoretical values.**

I. INTRODUCTION

The main difficulty in measuring the circuit parameters of microstrip discontinuities resides in the elimination of systematic errors introduced by coaxial-to-microstrip transitions. This problem can be avoided by testing discontinuities in a resonant microstrip ring which may be coupled very loosely to the test equipment.

Stephenson and Easter [1] and Douville and James [2] have demonstrated the resonant technique as applied to the characterization of rectangular bends. Groll and Weidmann [3] have evaluated impedance steps in a resonant ring.

The present short paper gives a comprehensive general analysis of a microstrip ring containing a reciprocal discontinuity. First it is shown how the Z parameters of a discontinuity are related to the change in resonant frequencies and Q factors of a microstrip ring it perturbs. Then the measurement technique is described, the accuracy of the method is discussed, and some measurements made on inductive metallic posts are presented.

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